



Accurately Estimating D3.1 Channel Capacity

A Technical Paper prepared for SCTE/ISBE by

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Introduction

The DOCSIS 3.1 specification fundamentally changes the nature of information delivery across the cable plant, and how the cable plant will be maintained and managed. The modulation order and FEC can be optimized based on actual plant conditions at individual devices. Devices which receive clean signals will utilize very efficient high-order-modulation across each of the subcarriers, devices with a degraded signal will use more robust modulation, all on the same channel. To manage this optimization the CMTS uses the concept of downstream OFDM profiles and upstream OFDMA profiles. D3.1 allows defining multiple profiles, each tuned to account for plant conditions experienced by a set of CMs.

Estimating downstream and upstream channel capacity was relatively straightforward for SC-QAM channels. But in D3.1, since the modulation orders of each subcarrier could be different and different across profiles, estimating the DOCSIS channel capacity is no longer simple. With multiple modulation profiles in use simultaneously, the capacity of the channel as seen by each CM may change instantaneously. The aggregate channel capacity calculation from a CMTS point of view with the different CMs using different profiles becomes more complicated and varies with which CMs and profiles are in use. How does one account for the NCP, FEC, PHY, MAC layer overhead and other variable factors in determining the effective channel throughput? How does profile definition, number of CMs, and heavy vs light traffic users on the channel affect throughput? The channel capacity affects how many subscribers can be assigned to use the same set of channels. It also affects traffic engineering and when an operator would need to split the node to increase available capacity. What is the reliable method for an operator to get a handle on the network capacity?

This paper will present a framework to calculate the D3.1 downstream and upstream channel capacity accurately and answer the above questions and considerations.





Downstream Capacity calculations

DOCSIS 3.1 introduces Orthogonal Frequency Division Multiplexing (OFDM) downstream signals and Orthogonal Frequency Division Multiple Access (OFDMA) upstream signals to achieve robust operation and provide more efficient use of the spectrum than previous DOCSIS versions.

The DOCSIS 3.1 system will have options of several split configurations that can be exercised based on traffic demand, services offered and the capability of the cable plant.

1. Band edges

DOCSIS 3.1 uses OFDM for downstream modulation. In the downstream direction, the cable system is assumed to have a pass band with a lower edge of either 54 MHz, 87.5 MHz, 108 MHz or 258 MHz, and an upper edge that is implementation-dependent but is typically in the range of 550 to 1002 MHz. Upper frequency edges extending to 1218 MHz, 1794 MHz and others are expected in future migrations of the plants. Within that pass band, analog or digital television signals in 6 MHz channels are assumed present on the standard, as well as other narrowband and wideband digital signals.

The CM supports a minimum of two independently configurable OFDM channels each occupying a spectrum of up to 192 MHz in the downstream. The demodulator in the CM supports receiving downstream transmissions up to at least 1.218 GHz and optionally support receiving downstream transmissions up to at least one or more of the following downstream upper band edges: 1.002 GHz, 1.218 GHz, 1.794 GHz.

1.1. Downstream Subcarriers

The OFDM downstream multicarrier system is composed of a large number of subcarriers that have either 25 kHz or 50 kHz spacing. These subcarriers are grouped into independently configurable OFDM channels each occupying a spectrum of up to 192 MHz in the downstream, totaling up to 7680 25 kHz subcarriers or up to 3840 50 kHz subcarriers; of which up to 7600 (25 kHz) or up to 3800 (50 kHz) active subcarriers spanning 190 MHz.

The encompassed spectrum of a 192 MHz downstream OFDM channel does not exceed 190 MHz. Therefore, the number of contiguous active subcarriers in a downstream OFDM channel does not exceed 3800 for 4K FFT and 7600 for 8K FFT. When configured for 4K FFT, the CMTS only uses subcarriers in the range 148 $\leq k \leq 3947$, where k is the spectral index of the subcarrier in the IDFT equation defining the OFDM signal. When configured for 8K FFT, the CMTS only uses subcarriers in the range 296 $\leq k \leq 7895$.

For D3.1 OFDM Channels there is at least 1 MHz of exclusion band between the spectral edge of a legacy SC-QAM channel and the center frequency of the nearest OFDM subcarrier and at least a 2 MHz exclusion band between any two adjacent asynchronous OFDM channels.





1.2. Symbol sizes

Parameter	Value		
Downstream master clock frequency	10.24	MHz	
Downstream Sampling Rate (fs)	204.8	MHz	
Downstream Elementary Period (Tsd)	1/(204.8 MHz)		
Channel bandwidths	24 MHz 192 MHz		
IDFT size	4096	8192	
Subcarrier spacing	50 kHz	25 kHz	
FFT duration (Useful symbol duration) (Tu)	20 µs	40 µs	
Maximum number of active subcarriers in	3800	7600	
signal (192 MHz channel -190 MHz used)			
Maximum spacing between first and last	190 MHz		
active subcarrier			

Table 1 - D3.1 Downstream OFDM Parameters

1.3. Cyclic Prefix

A segment at the end of the IDFT output is prepended to the IDFT, and this is referred to as the Cyclic Prefix (CP) of the OFDM symbol. There are five possible values for the length of the CP for the D3.1 Downstream and the choice depends on the delay spread of the channel, a longer delay spread requires a longer cyclic prefix.



Figure 1 - Cyclic Prefix Concept

Cyclic Prefix (us)	CP samples
0.9375 μs	(192 * Tsd)
1.25 μs	(256 * Tsd)
2.5 μs	(512 * Tsd)
3.75 µs	(768 * Tsd)
5 μs	(1024 * Tsd)

1.4. Modulation Orders

The D3.1 Downstream modulation orders supported are 16-QAM, 64-QAM, 128-QAM, 256-QAM, 512-QAM, 1024-QAM, 2048-QAM, and 4096-QAM and optionally support 8192-QAM and 16384-QAM.





1.5. Pilots & PLC

Continuous pilots are pilots that occur at the same frequency location in every OFDM symbol. The PHY Link Channel (PLC) resides in a contiguous set of subcarriers in the OFDM channel. These subcarriers occupy the same spectral locations in every symbol. Continuous and scattered pilots change from symbol to symbol. Since overlapping pilots are treated as continuous pilots, the number of scattered pilots' changes from symbol. For bit loading, continuous pilots and the PLC are treated in the same manner as excluded subcarriers

The PLC is comprised of 8 OFDM subcarriers in every OFDM symbol when using 4K FFT OFDM (i.e., a subcarrier spacing of 50 kHz) and 16 OFDM subcarriers when using 8K FFT OFDM (i.e., a subcarrier spacing of 25 kHz).

Using the following notation

- N: The total number of subcarriers in the OFDM symbol, equaling either 4096 or 8192
- NC: The number of continuous pilots in an OFDM symbol
- *NS*: The number of scattered pilots in an OFDM symbol
- NE: The number of excluded subcarriers in an OFDM symbol
- *NP*: The number of PLC subcarriers in an OFDM symbol
- ND: The number of data subcarriers in an OFDM symbol

The values of *N*, *NC*, *NE* and *NP* do not change from symbol to symbol for a given OFDM template; the values of *NS* and *ND* change from symbol to symbol. The following equation holds for all symbols: $N = NC + NS + NE + NP + ND \qquad ---ean(1)$

The value of N is 4096 for 50 kHz subcarrier spacing and 8192 for 25 kHz subcarrier spacing. (NS + ND) is a constant for a given OFDM template. Therefore, although the number of data subcarriers (ND) and the number of scattered pilots (NS) in an OFDM symbol changes from symbol to symbol, the sum of these two numbers is invariant over all symbols.

There are eight predefined continuous pilots around the PLC. In addition, there are a number of continuous pilots distributed as uniformly as possible over the entire OFDM spectrum. The number of Continuous Pilots is given using the following formula:

Number of Continuous Pilots = $min(max (8, ceil(M * \Box(Fmax-Fmin)/190), 120) ---eqn(2)$

*Fm*ax refers to frequency in MHz of the highest frequency active subcarrier and *Fm*in refers to frequency in MHz of the lowest frequency active subcarrier of the OFDM channel. Per the equation, the number of continuous pilots is linearly proportional to the frequency range of the OFDM channel. The minimum number of continuous pilots defined using the PLC cannot be less than 8, and the maximum number of continuous pilots defined using the PLC cannot exceed 120. Therefore, the total number of continuous pilots, including the predefined ones(PLC), will be in the range 16 to 128, both inclusive, see eqn(2). The value of M in equation as a parameter that can be adjusted by the CMTS with $120 \ge M \ge 48$, the typical value proposed for M is 48.

The main purpose for scattered pilots is the estimation of the channel characteristics for the purpose of equalization. There are two scattered pilot patterns, one for 4K FFT and one for 8K FFT. Although these pilots occur at different frequency locations in different OFDM symbols, the patterns repeat after every





128 OFDM symbols; in other words, the scattered pilot pattern has a periodicity of 128 OFDM symbols along the time dimension.

1.6. DS Capacity Calculations

Below are some snippets of Python code which was used in the capacity calculation for the DOCSIS 3.1 channels. The code shows how to calculate the Number of Effective subcarriers in a channel, given certain input settings.

DSUpperBandEdge = DSLowerBandEdge + DSOccupiedSpectrum	
DSNumFFTPoints = (DSSamplingRate *1000)/ DSSubcarrierSpacing	
DSSymbolPeriod_usec = 1000/DSSubcarrierSpacing	
DSCyclicPrefix_usec = DSCyclicPrefix/DSSamplingRate	
DSActualSymbolPeriod_usec = DSSymbolPeriod_usec + DSCyclicPrefix_usec	
DSSymbolEfficiency = 100 *DSSymbolPeriod_usec/DSActualSymbolPeriod_usec	
DSModulatedSubcarriers = (DSOccupiedSpectrum - DSGuardBand – DSExcludedBand)	
* 1000 / DSSubcarrierSpacing	
if DSSubcarrierSpacing == 50:	
DSNumPLCSubcarriers = 8	
else:	
DSNumPLCSubcarriers = 16	
DSNumContPilots = min(max(8, ceil(DSPilotDensity_M * DSOccupiedSpectrum /190)),	
	120) + 8
DSNumScatteredPilots = ceil((DSModulatedSubcarriers -DSNumPLCSubcarriers)/128)	
DSEffectiveSubcarriers = DSModulatedSubcarriers - (DSExcludedSubcarriers +	
DSNumPLCSubcarriers * DSNumFFTBlocks +	
DSNumContPilots + DSNumScatteredPilots)	

2. Downstream FEC

A Downstream codeword (FEC Frame) will be of the size 16,200 bits; this is using the Low Density Parity Codes (LDPC) at a code rate of 8/9. In a D3.1 OFDM channel an FEC Codeword can include mixed modulation. (A mixed-modulation codeword belongs to a profile that does not use the same modulation constellation for all subcarriers of the OFDM symbol.) Codeword shortening is also supported.

Table 3 - FEC	coding	parameters
---------------	--------	------------

LDPC Code Rate	BCH Uncoded Block Size	BCH Coded	LDPC Uncoded	LDPC Coded Block
8/9	14,232	14,400	14,400	16,200

The downstream LDPC codeword shown in Figure 2 is referred to as (16200, 14400). This means that a full codeword is 2025 bytes (16200 bits) that are divided into 225 bytes (1800 bits) of parity and 1800 bytes (14400 bits) of LDPC payload. That payload is further divided into 21 bytes (168 bits) of BCH parity, a 2 byte fixed header, and a variable 1777 byte maximum payload for DOCSIS frames. When the FEC codeword is shortened, only the DOCSIS payload shrinks. All other fields remain the same size.

Codeword shortening is used for creating shortened codewords when there is insufficient data to fill complete codewords or to achieve strong burst noise protection. Codeword shortening is accomplished by





shortening the uncoded block size (14232). Note that the number of parity bits remains the same; there is no shortening of the parity bits either in the BCH or in the LDPC. When a shortened codeword is needed, the CMTS prepends zero bits to the data, do the BCH encoding and LDPC encoding and removes the appended zero bits.



Figure 2 - DS Downstream Codeword header and payload

In the CW Header, the PDU Pointer (11 bits) points to the first byte of the first DOCSIS frame that starts in the payload. A value of zero points to the byte immediately following the codeword header.

2.1. Calculated Number of DS FEC codewords

The CMTS can make a choice to service traffic from each Downstream OFDM Profile in a round-robin fashion. It can do this by sending packets destined to CMs assigned to each profile during a specific window. This window can be on a symbol to symbol basis. i.e. say if there are 4 active profiles: A, B, C, D, the CMTS can choose to send traffic to profile A on a certain symbol and profile B, C, D on each subsequent symbol and cycle through again. It can also decide to make that window larger and choose to service each profile for a longer period of time say 2, 4, or 8 symbols at a time. The CMTS can also send codewords from multiple profiles on the same symbol, this case has not been explicitly calculated in this paper, as the main goal is to try and calculate effective throughput under fully loaded conditions. However, in this case one can surmise that switching between profiles can drop the FEC efficiency further, as there may be present multiple shortened codewords.

The following graphs show the number of full(long) & shortened codewords which will fit into DS symbols, if the CMTS chooses 1, 2, 4, or 8 symbols for packets from each profile. The graphs also show the variation of the number of codewords across increasing Channel Bandwidth. Additionally, it also shows the PHY efficiency of those channel across a symbol.







Figure 3 - Number of DS FEC Codewords per symbol vs Channel BW (1 symbol per profile)



Figure 4 - Number of DS FEC Codewords per symbol, vs Channel BW (2 symbols per profile)







Figure 5 - Number of DS FEC Codewords per symbol, across Channel BW (4 symbols per profile)



Figure 6 - Number of DS FEC Codewords per symbol, across Channel BW (8 symbols per profile)





3. Downstream Rate and efficiency

3.1. Covergence Layer Overhead

Detecting where the next codeword begins in an OFDM symbol can be difficult because more than one codeword may map into one OFDM symbol, the number of codewords per OFDM symbol may not be an integer, a codeword can overflow from one OFDM symbol to another, and the codeword could be shortened. Therefore, the transmitter must convey to the receiver all of the locations where a new codeword begins within an OFDM symbol. When the data codewords are mapped to subcarriers within a symbol, a pointer is needed to identify where a data codeword starts. This is known as the Next Codeword Pointer (NCP).

There are a variable number of NCP message blocks (MBs) on each OFDM symbol. To make sure that all subcarriers are used without reserving empty NCP MBs, the mapping of the NCP occurs in the opposite direction of the mapping for data. The relationship of NCP message blocks to the data channel is shown in Figure 7 (the last NCP MB is always a CRC-MB). These Next Codeword Pointers (NCPs) are encoded using the NCP FEC encoder, as defined in [D3.1 PHY] and are appended to the OFDM symbols. NCP subcarriers are modulated using QPSK, 16-QAM, or 64-QAM and this modulation is signaled by the PLC.



Figure 7 - NCP Examples

3.2. MAC Layer Management Messaging Overhead

The below table is an estimate of the packet sizes and data rate consumed by some of the common periodic MAC Management Messages (MMM): Upstream Channel Descriptor (UCD), MAC Domain Descriptor (MDD), OFDM Channel Descriptor (OCD), Downstream Profile Descriptor (DPD), Upstream Bandwidth Allocation Map (MAP), Time Synchronization (SYNC)





	DPD	OCD (On PLC only)	MDD	UCD	MAP	Sync
Total bits (Average msg size * number of messages)	1156 (289*4 profiles)	306 (306 * 1 OFDM channel)	1210	459	594 (66 * 9) (8 SC QAM and 1 OFDMA Channel)	34
Periodicity in milliseconds	500	200	2000	2000	2	200
Average Data rate (Bits per second)	18,496	12,240	4,840	1,836	2,376,000	1,360

Table 4 - D3.1 Downstream MMM Overhead

Based on this table we can see that MDDs, DPDs, OCDs, UCDs, Syncs, with above average sizes of messages only add up to ~38 kbps. With the addition of MAPs, the MMM overhead adds up to 2.4 Mbps.

Give that in a D3.1 OFDM channel we are talking of channel capacity from ~200 Mbps(24 MHz) to upwards of 1.7 Gbps (192 MHz), the MMM overhead may be considered insignificant in the data Rate Calculation.

3.3. Calculations for Downstream rate & PHY efficiency

Below are some snippets of Python code which was used in the capacity calculation for the DOCSIS 3.1 channels. The code shows how to calculate the data rate of a channel (assuming one profile) and the downstream PHY efficiency.

This step calculates the number of subcarriers consumed by each NCP message block.

NCPBitsperMB = 48 SubcarriersPerNCPMB = NCPBitsperMB/NCPModulationOrder

This step calculates the number of data bits available in each OFDM DS Symbol. Also if the CMTS services the profile for multiple symbols, then the number of available bits are multiplied by the number of symbols.

NumBitsinDataSubcarriers = DSEffectiveSubcarriers * DSAvgModulationOrder **if** numSymbolsPerProfile > 1: NumBitsinDataSubcarriers = NumBitsinDataSubcarriers * numSymbolsPerProfile

This step calculates the number of full codewords available on the DS symbols for that burst of traffic on a particular profile (one or more OFDM DS Symbol, see above step).

DSLDPC_FEC_CW = CWSize, Infobits, Parity, BCH, CWheader DSLDPC_FEC_CW = [16200, 14216, 1800, 168, 16]

NumFullCodewords = math.floor(NumBitsinDataSubcarriers/DSLDPC_FEC_CW[0])

This step calculates the length of a shortened codeword if possible. It does this by finding the number of bits left in the symbol, by taking away the number of bits needed for the NCP MBs for those full Codewords and the CRC NCP MB,





```
NumNCPMBs = NumFullCodewords + ceil(1*numSymbolsPerProfile)
```

```
EstimateShortendedCWSize =

((numSymbolsPerProfile * DSEffectiveSubcarriers -

((NumNCPMBs + 1) * SubcarriersPerNCPMB)) * DSAvgModulationOrder) -

(DSLDPC_FEC_CW[0] * NumFullCodewords)

if EstimateShortendedCWSize > 0:

ShortendedCWData =

EstimateShortendedCWSize - (DSLDPC_FEC_CW[2] -DSLDPC_FEC_CW[3]-DSLDPC_FEC_CW[4])

else:

ShortendedCWData = 0
```

This step calculates the number of data bits per symbol and then uses the actual symbol period to calculate the effective data rate of the channel. The Downstream PHY efficiency is calculated by dividing by the Downstream occupied spectrum.

```
totalDataBits = (NumFullCodewords*DSLDPC_FEC_CW[1]) + ShortendedCWData
RateacrossWholeChannelGbps = totalDataBits/
(DSActualSymbolPeriod_usec*numSymbolsPerProfile*1000)
DSPHYEfficiency = RateacrossWholeChannelGbps*pow(10,3)/(DSOccupiedSpectrum)
```

The graphs in Figure 8, Figure 9 show the effective data rates of the OFDM channel vs. increasing channel bandwidth. Each curve indicates the number of continuous symbols the CMTS uses to send data for a particular profile. The red curve is 1 DS symbol per profile, blue is 4 DS symbols per profile, and green is 8 DS symbols per profile.



The graph in Figure 10, below shows the effective data rates and the PHY Efficiency of a DS OFDM channel (192 MHz) vs increasing Cyclic prefix size. The graph in Figure 11, shows the effective data





rates & the PHY Efficiency of the OFDM channel(192 MHz, 512 CP), vs increasing modulation order for the profile.







The graph in Figure 12 shows the effective data rates of the OFDM channel, vs increasing Modulation order for the profile, and changing cyclic prefix values (each curve is a different CP setting, increasing CP reduces the effective data rate)



Figure 12 - D3.1 Downstream Rates vs Modulation order





4. Multiple profiles

The DOCSIS 3.1 OFDM introduces multiple modulation orders within a channel. Each of the subcarriers in the downstream OFDM channel can be configured to use a different modulation order. This allows the CMTS to optimize the downstream transmissions across the wide frequency band (192 MHz) of the channel. The specific choice of modulation order for each subcarrier is communicated to the CMs in the form of a downstream modulation profile, which allows the CMs to interpret and demodulate the signal.

A modulation profile consists of a vector of bit-loading values, an integer value for each active subcarrier in the downstream channel. Since the modulation orders range from 16-QAM to 16384-QAM, the range of bit-loading values is from 4 to 14 (skipping 5). However, it is expected that very low bit-loading values, 7 or less, will be used very infrequently since most plants support 256 QAM today.

The CMTS uses a "Profile A" that is the lowest common denominator profile, i.e. Profile A is able to be successfully received by all CMs in the Service Group. Profile A is essentially the lowest modulation across all CMs and subcarriers in the channel. It can then generate up to 15 additional modulation profiles, which are communicated to the Service Group. Each CM can be assigned up to four modulation profiles, including Profile A (used for broadcast frames), an optimized profile for the CM's unicast traffic, and possibly two additional profiles that could be used for multicast traffic. Since the number of CMs in the service group, using that Downstream channel is expected to be larger than 15 in the majority of cases, each profile is expected to be used by a group of CMs that have similar channel characteristics.

Calculating the effective channel capacity when there are multiple profiles active on a CMTS is complicated. The amount of data traffic used by various CMs can vastly differ based on the various subscribers. Each CM can be assigned a different set of profiles that are optimized for it. By combing the profile information and CM data usage one can start estimating the current effective channel capacity. As the data usage of the various CMs changes, the data sent on each of the profiles changes, effectively changing the data rate of channel This can occur on a symbol to symbol basis. This would probably be best modeled and estimated using a Monte Carlo simulation. That effort could be the topic of another paper.

For simpler estimates which will help us approximate the data rate capacity, one can average out the profile modulation order and weight it across the number of CMs assigned to a particular profile to work out an approximate channel capacity. This assumes each CM is receiving the same amount of traffic. One could also calculate this average with different assumptions on traffic from each CM, their associated profiles and weighting it appropriately.

Using the Profile Management Algorithms defined in [PMA-2016], one can find an optimal set of profiles for use on a channel for a set of CMs. The data one needs to have is the RxMER per subcarrier across the channel for each CM. Based on a set of RxMER Data observed in CMs in a D3.1 trial below are some of the potential profile definitions created by using the [PMA-2016] algorithms.

This figure shows the profile definition of 8 different DS profiles (7 individual profiles + Profile A which is the least common denominator profile), for a group of 121 CMs. It shows the number of subcarriers in each profile that have a specific bit loading e.g. Profile 2 (green) has 27 CMs assigned to it, and has an average Modulation Order (bit-loading) of 11.98, or 4096 QAM across most subcarriers and a few subcarriers at a level below that. Other profiles are also a combination of different modulation orders on different subcarriers.







Figure 13 - Sample set of D3.1 Profile definitions

Now to estimate the D3.1 capacity, assuming all CMs receive equal amounts of traffic, the average modulation order across all the CMs would end up being the weighted average of the profile average modulation order and the number of CMs using it. For the set of CMs/profiles in the Figure 13 that weighted average modulation order (bit-loading) would be 10.75 bits, which will yield a mean channel capacity ~1.6245 Gbps (with smallest CP value). The instantaneous capacity will vary depending on which set of CMs and profiles are active.

Upstream Capacity calculations

DOCSIS 3.1 uses OFDMA (orthogonal frequency-division multiple access) for upstream modulation. OFDMA is a multi-user version of OFDM, and assigns subsets of subcarriers to individual CMs. The upstream OFDMA parameters are derived from the downstream parameters, and described below

5. Band edges

In the upstream direction, the cable system may have a 5-42 MHz, 5-65 MHz, 5-85 MHz, 5-117, 5-204 MHz or pass bands with an upper band edge beyond 204 MHz. Analog and digital television signals in 6 MHz channels may be present, as well as other signals. A D3.1 CM supports one or more of the following upstream upper band edges, (as long as one is 85 MHz or greater): 42 MHz; 65 MHz, 85 MHz, 117 MHz, and/or 204 MHz. DOCSIS 3.1 Network supports a minimum of two independently configurable OFDMA channels each occupying a spectrum of up to 95 MHz in the upstream. A DOCSIS 3.1 network is capable of receiving 192 MHz of upstream active channels when operating with the 204 MHz upstream upper band edge. In DOCSIS 3.1 upstream mode the CM is capable of transmitting OFDMA channels and legacy SC-QAM channels at the same time (as controlled by the CMTS). There are no legacy SC-QAM channels above a frequency of 85 MHz.

5.1. Upstream Subcarriers

The OFDMA upstream multicarrier system is also composed of either 25 kHz or 50 kHz subcarriers. In the upstream, the subcarriers are grouped into independently configurable OFDMA channels each of up to 95 MHz encompassed spectrum, totaling 3800 25 kHz spaced subcarriers or 1900 50 kHz spaced subcarriers or 1920 50 kHz spaced subcarriers. Many parameters of these channels can be independently configured thereby optimizing configuration based on channel conditions

The encompassed spectrum of an upstream OFDMA channel does not exceed 95 MHz. Therefore, the number of contiguous active subcarriers in an upstream OFDMA channel are1900 for 2K FFT and 3800





for 4K FFT. When configured for 2K FFT, the CMTS only uses subcarriers in the range 74 $\leq k \leq 1973$, where k is the spectral index of the subcarrier in the IDFT equation defining the OFDMA signal. When configured for 4K FFT, the CMTS MUST only use subcarriers in the range 148 $\leq k \leq 3947$.

5.2. Symbol sizes

Parameter	Value		
Upstream Sampling Rate (fsu)	102.4 MHz		
Upstream Elementary Period Rate (Tsu)	1/102.4 MHz		
Channel bandwidths	10 MHz,, 95	6.4 MHz,, 95	
	MHz	MHz	
IDFT size	2048	4096	
Subcarrier spacing	50 kHz	25 kHz	
FFT duration (Useful symbol duration) (Tu)	20 µs	40 µs	
Maximum number of active subcarriers in	1900	3800	
signal (for 95 MHz channel) 1900 3800			

Table 5 - D3.1 Upstream OFDMA Parameters

5.3. Cyclic Prefix

A segment at the end of the IFFT output is prepended to the IFFT output; this is referred to as the Cyclic Prefix (CP) of the OFDM symbol. The addition of a cyclic prefix enables the receiver to overcome the effects of inter-symbol-interference caused by micro-reflections in the channel.

Cyclic Prefix (us)	CP samples
0.9375 μs	(96 * T _{su})
1.25 μs	(128 * T _{su})
1.5625 μs	(160 * T _{su})
1.875 μs	(192 * T _{su})
2.1875 μs	(224 * T _{su})
2.5 μs	(256 * T _{su})
2.8125 μs	(288 * T _{su})
3.125 μs	(320 * T _{su})
3.75 µs	(384 * T _{su})
5.0 μs	(512 *Tsu)
6.25 μs	(640 *Tsu)

 Table 6 - D3.1 Upstream Cyclic Prefix Parameters

5.4. Upstream Modulation orders

The DOCSIS 3.1 network supports BPSK, QPSK, 8-QAM, 16-QAM, 32-QAM, 64-QAM, 128-QAM, 256-QAM, 512-QAM, 1024-QAM, 2048-QAM, and 4096-QAM for subcarriers of upstream OFDMA channels. BPSK is used for pilots and complementary pilots only, and not used for data subcarriers.





5.5. OFDMA Frames

Upstream transmission uses OFDMA frames. Each OFDMA frame is comprised of a configurable number of OFDM symbols, K. Several transmitters may share the same OFDMA frame by transmitting data and pilots on allocated subcarriers of the OFDMA frame.

There are several pilot patterns as described below. The structure of an OFDMA frame is depicted in Figure 14. The upstream spectrum is divided into groups of subcarriers called minislots. Minislots have dedicated subcarriers, all with the same modulation order ("bit loading"). A CM is allocated to transmit one or more minislots in a Transmission Burst. The modulation order of a minislot, as well as the pilot pattern to use may change between different transmission bursts (see Figure 15) and are determined by a transmission profile.



Figure 14 - Example Minislot Layout









Figure 15 - Grants across Minislots / Tx Burst Layout

Figure 16 - Edge and Body Minislots in a Transmission Burst

5.6. Pilots & Minislots

DOCSIS 3.1 specifies two minislot sizes by specifying the number of subcarriers per minislot. There are 8- and 16-subcarrier minislots. A minislot is always 400 kHz wide (25KHz subcarrier *16, or 50KHz subcarrier *8). Two types of minislots are defined for each minislot size: edge minislots and body minislots. An edge minislot is the first minislot in a transmission burst, and body minislots are used for all other minislots in a transmission burst, see Figure 16. An edge minislot is used for the first minislot of an OFDMA frame that is not a zero valued minislot and also for the first minislot after an exclusion band or after one or more contiguous skipped subcarriers or after a zero valued minislot.

Each minislot is comprised of pilots(P), complementary pilots(CP), and data subcarriers. Pilots are used by the CMTS receiver to adapt to channel conditions and frequency offset. Pilots are subcarriers that do not carry data, Instead, it encodes a pre-defined BPSK symbol known to the receiver. DOCSIS 3.1 also specifies complementary pilots which are subcarriers that carry data, but with a lower modulation order than other data subcarriers in the minislot. If the modulation order used for data in the minislot is M, the complementary pilots are used with modulation order equal to the maximum between M-4 and 1 (BPSK). For example, if the bit loading in a minislot is 12, Complementary Pilots use 8 bits.

For each minislot size, seven pilot patterns are defined. Pilot patterns differ by the number of pilots in a minislot, and by their arrangement within the minislot. The different pilot patterns enable the CMTS to optimize its performance (physical layer rate and pilot overhead) according to different loop conditions and variations of SNR with frequency. Each pilot pattern defines edge and body minislots.







Figure 18 - Pilot Patterns 8-14 (16 subcarrier minislots)

Below are some snippets of Python code which was used in the capacity calculation for the DOCSIS 3.1 channels. The code shows how to calculate the data capacity of a minislot, given certain input settings, such as the OFDMA frame width in symbols (K), num of Subcarriers (Q), body vs Edge Mini slot, and the pilot pattern and complementary pilot pattern.

Pattern Number, minislotsubcarriersQ, Body0/Edge1, Num Pilots, Num CP minislotpatterns = [[1, 8, 0, 2, 2], [2, 8, 0, 4, 2], [3, 8, 0, 8, 2], [4, 8, 0, 16, 2], [5, 8, 0, 1, 1], [6, 8, 0, 2, 1], [7, 8, 0, 4, 1], [1, 8, 1, 4, 4], [2, 8, 1, 6, 4], [3, 8, 1, 10, 4], [4, 8, 1, 16, 4], [5, 8, 1, 2, 2], [6, 8, 1, 3, 2], [7, 8, 1, 5, 2], [8, 16, 0, 2, 2], [9, 16, 0, 4, 2], [10, 16, 0, 8, 2],





[11, 16, 0, 16, 2], [12, 16, 0, 1, 1], [13, 16, 0, 2, 1], [14, 16, 0, 4, 1], [8, 16, 1, 4, 4], [9, 16, 1, 6, 4], [10, 16, 1, 10, 4], [11, 16, 1, 18, 4], [12, 16, 1, 2, 2], [13, 16, 1, 3, 2], [14, 16, 1, 5, 2],]

def MinislotCapcity(K, Modulationorder, pattern_index): """ function for the Minislot calculation""" **global** minislotpatterns

Q = minislotpatterns[pattern_index][1] cp_modulation_order = max(Modulationorder - 4, 1) subcarriers = K * Q - minislotpatterns[pattern_index][3] - minislotpatterns[pattern_index][4] mscapacity = Modulationorder * subcarriers + cp_modulation_order * minislotpatterns[pattern_index][4]

return mscapacity

The following Figure 19 below show the minislot capacity across various parameters. The first sub-plot shows the variation of minislot capacity(K=16), across the different Pilot patterns. It shows 4 lines, two pairs for the 8 & 16 subcarrier minislots, each one denoting body and edge. The edge minislot is a smidge lesser in data capacity compared to the Body minislot

The second sub-plot shows the variation of the Minislot capacity (Q=16, pattern 13) as the K increases from 6 to 36 across, and the modulation order of the minislot increases from 2 to 12 bits bottom to top.







Figure 19 - MiniSlot Capacity : Across Pilot Patterns, Minislot Width(K symbols), and Modulation order

6. FEC

The grant indicates which minislots are assigned to a given burst and which upstream profile is to be used. The CM and CMTS use this information to determine the total number of bits in the grant which are available to be used for FEC information or parity.

Code	LDPC Code Rate	Codeword size in bits (Ni)	Information bits (Ki)	Parity bits (Pi)
Long	8/9 = 89%	16200	14400	1800
Medium	28/33 = 85%	5940	5040	900
Short	3⁄4 = 75%	1120	840	280





6.1. Shortened codewords

Based on the size of the grant obtained from the CMTS, The US FEC codeword creation algorithm follows the procedure as depicted in the Figure 20. The [D3.1 PHY] spec in section 7.4.3.1.1 FEC Codeword Selection Algorithm, shows an implementation of this procedure in Matlab code.



Figure 20 - US CW creation Algorithm

Based on the implementation of the above algorithm, the graphs in Figure 21, show the number of code words of each kind (Short, Shortened Short, Medium, Shortened Medium, Long, Shortened Long) in different colors, as the grant size (in bits) varies. The graph also shows the FEC efficiency of the grant, see Figure 22.





FEC Efficiency



Figure 21 - Number of US FEC Codewords over grant sizes, FEC Efficiency



Figure 22 - Effective US FEC Efficiency vs grant size





7. Calculations for Upstream rate & PHY efficiency

Below are some snippets of Python code which was used in the capacity calculation for the DOCSIS 3.1 channels. The code shows how to calculate the data rate of a channel and the downstream PHY efficiency. The assumption here is an average modulation order calculated across all the minislots for the whole data profile or IUC (interval usage code).

This step calculates the active bandwidth, the symbol period, and the total frame duration.

```
USUpperBandEdge = USLowerBandEdge + USOccupiedSpectrum

USActiveBW = USOccupiedSpectrum – USExcludedSpectrum

USCyclicPrefix_usec = USCyclicPrefix / USSamplingRate

USNumFFTPoints = 1000 * USSamplingRate / USSubcarrierSpacing

USSymbolPeriod_usec = 1000 / USSubcarrierSpacing

USActualSymbolPeriod_usec = UsSymbolPeriod_usec + USCyclicPrefix_usec

USSymbolEfficiency = 100 * UsSymbolPeriod_usec / USActualSymbolPeriod_usec

USFrameDuration_usec = USMinislotSymbolSK * USActualSymbolPeriod_usec
```

This step calculates the number of subcarriers in the Minislot (Q), and prepares to calculate the excluded subcarriers

```
if USSubcarrierSpacing == 25:
    USMinislotSubcarriersQ = 16
    kNBI = 3
else:
    USMinislotSubcarriersQ = 8
    kNBI = 2
```

This step calculates the number of excluded subcarriers, uses the exclusion zones, the number of grants, to figure out the number of minislots

```
UsTotalSubcarriers = 1000 * USOccupiedSpectrum / USSubcarrierSpacing
USExcludedSubcarriers = (1000 * (USExcludedSpectrum + USGuardBand) /
USSubcarrierSpacing) + (USExcludedNBI * kNBI)
UsNumofExclSpectrumGaps = USExcludedNBI + USNumContlegacy
USActualSignalSubcarriers = UsTotalSubcarriers - USExcludedSubcarriers
USTempNumMinislots = math.floor(USActualSignalSubcarriers /
USMinislotSubcarriersQ)
UsminislotEfficiency = (USActualSignalSubcarriers–UsNumofExclSpectrumGaps * 4) /
USActualSignalSubcarriers
USNumMinislots = round(UsminislotEfficiency * USTempNumMinislots)
USNumofEdgeMinislots = USNumGrantsInProfile + USAddnIEdgeMiniSlot
USNumofBodyMinislots = USNumMinislots – USNumofEdgeMinislots
```





This step calculates the appropriate pilot pattern, and then the total US capacity of an US OFDM frame, by calculating the minislot capacity for all the Body and Wdge Minislots.

```
local_USPilotPattern = USPilotPattern-1 # convert frm spec index to array index
if USMinislotSubcarriersQ == 16:
    local_USPilotPattern = local_USPilotPattern + 7

usCapacity = (
    (USNumofBodyMinislots * MinislotCapcity(USMinislotSymbolsK, USProfileModOrder,
    local_USPilotPattern)) +
    (USNumofEdgeMinislots * MinislotCapcity(USMinislotSymbolsK, USProfileModOrder,
    local_USPilotPattern + 7))
)
```

avgMinislotCapacity = usCapacity / USNumMinislots

This step calculates the channel rate using the US minislot capacity number and the actual symbol period, and the PHY efficient with FEC overhead

```
ProfileRate_Mbps = (usCapacity /
(USMinislotSymbolsK * USActualSymbolPeriod_usec))
usPHYEfficiency = ProfileRate_Mbps / USActiveBW
usPHYEfficiency_w_fec_time_overhead = usPHYEfficiency *
getCWSizesforEfficiency(usCapacity)[0] *
(USSymbolEfficiency / 100)
```

The graphs in Figure 23, Figure 24, show the effective data rates and PHY Efficiency of the OFDMA channel vs. increasing channel bandwidth.



Figure 23 - US Channel Capacity across Channel BW







Figure 24 - US PHY Efficiency Bits per hz across Channel BW

The graph in Figure 25 shows the effective data rates and the PHY Efficiency of a US OFDM channel (96 MHz) vs increasing Cyclic prefix size.



Figure 25 - US PHY Efficiency Bits/s/hz & Channel capacity vs CyclicPrefix us

The graph in Figure 26 below shows the effective data rates of the OFDMA channel, vs increasing Modulation order for the profile.







Figure 26 - US PHY Efficiency Bits/s/hz & Channel capacity vs Modulation order

The graph in Figure 27 below shows the effective data rates & the PHY Efficiency of a US OFDMA channel (96 MHz) vs increasing grant size. Each curve is for different values of K ranging from 6 to 36



Figure 27 - US PHY Efficiency Bits/s/hz & Channel capacity vs Number of grants

The graph in Figure 28 shows the effective data rates and the PHY Efficiency of a US OFDMA channel (96 MHz) vs increasing Cyclic prefix size.







Figure 28 - US PHY Efficiency Bits per hz across Channel BW

8. Multiple profiles

Just like in the DS OFDM channel with DS modulation profiles, the US OFDMA channel has support for multiple modulation orders within an IUC (US Profile). All

The DOCSIS 3.1 OFDMA introduces multiple modulation orders within an Upstream channel. Each of the minislots in the upstream OFDMA channel can be configured to use a different modulation order. (A minislot always uses the same modulation order.) This allows the CMTS to optimize the upstream transmissions across the wide frequency band (95 MHz) of the channel. The specific choice of modulation order selected for each minislot is communicated to the CMs in the form of an upstream IUC/modulation profile, which allows them to interpret and modulate the signal.

A modulation profile consists of a vector of bit-loading values, an integer value for each minislot in the upstream channel. Since the modulation orders range from QPSK to 4096-QAM, the range of bit-loading values is from 2 to 12; however, it is expected that very low bit-loading values, 4 or less, will be used very infrequently since most plants support 16 or 64 QAM today.

The CMTS uses IUC 13 which is the lowest common denominator profile, able to be successfully used by all CMs in the Service Group, mainly before registration. CMTS can define additional IUCs (Data Profile IUCs: 5, 6, 9, 10, 11, 12, and 13) which are communicated to the Service Group. Each CM can be assigned up to two IUCs, Since the number of CMs in the Service Group is expected to be larger than 6 in the majority of cases, each IUC is expected to be used by a group of CMs that have similar channel characteristics.





Now calculating the effective channel capacity when there are multiple IUCs active on a CMTS starts getting complicated. The amount of data traffic used by various CMs can vastly differ based on the subscribers. Each CM can be assigned a different set of IUCs, optimized for it. By combing the IUC information and CM data usage one can start estimating the current effective channel capacity. As the data usage of the various CMs changes, the data sent on each of the profiles changes, effectively changing the data rate of channel, on a frame to frame basis. Again, like in the downstream,, this would probably be best modeled and estimated using a Monte Carlo simulation.

For simpler estimates which will help us approximate the data rate capacity, one can average out the IUC modulation order and weight it across the number of CMs assigned to a particular IUC to work out an approximate channel capacity. This assumes each CM is transmitting the same amount of traffic. One could also weight this average with different assumptions on traffic from each CM and their IUCs

Conclusion

Estimating downstream and upstream channel capacity in D3.1 gets complicated since the modulation orders of each subcarrier could be different and different across profiles/IUCs. This paper presents the details on the elements/calculations needed to compute the effective channel capacity of a DS OFDM channel or an US OFDMA channel. For the downstream this includes accounting for pilots, NCPs, FEC overhead of Long or shortened codewords, etc. For the upstream this includes accounting for pilots, complementary pilot patterns, Minislot sizes, cyclic prefix, FEC overhead of short medium or long and their shortened versions etc. The paper presents in graphical form many of the expected capacity numbers for the D3.1 downstream and upstream channel

Default Parameters for calculations

[DownstreamChannel]
DSOccupiedSpectrum = 192
DSLowerBandEdge = 678
DSGuardBand = 2
DSExcludedBand = 2
DSNumFFTBlocks = 1
DSAvgModulationOrder = 12
DSSamplingRate = 204.8
DSSubcarrierSpacing = 50
DSCyclicPrefix = 512
DSWindowing = 128
DSPilotDensity_M = 48
DSExcludedSubcarriers = 20
NCPModulationOrder = 6

[UpstreamChannel]
USOccupiedSpectrum = 96
USLowerBandEdge = 12
USSamplingRate = 102.4
USSubcarrierSpacing = 25
USPilotPattern = 8
USCyclicPrefix = 192
USWindowing = 128
USMinislotSymbolsK = 36
USNumContlegacy = 1
USExcludedSpectrum = 0
USGuardBand = 1
USExcludedNBI = 0
USAddnlEdgeMiniSlot = 0
USProfileModOrder = 10
USNumGrantsInProfile = 38





Abbreviations

bps	bits per second
СМ	Cable Modem
CMTS	Cable Modem Termination System
СР	Cyclic Prefix
DOCSIS	Data-Over-Cable Service Interface Specifications
D3.1	DOCSIS version 3.1 specification, Networks, equipment
DS	Downstream
DPD	Downstream Profile Descriptor
IUC	Interval Usage Code (US Profile)
IDFT	Inverse Discrete Fourier Transform
FEC	forward error correction
FFT	Fast Fourier Transform
HFC	hybrid fiber-coax
Hz	hertz
LDPC	Low-Density Parity Check
MAC	Media Access Control Layer
MAP	Bandwidth Allocation Map, MAC Management Message that the
	CMTS uses to allocate transmission opportunities to CMs
MB	Message Block.
MDD	MAC Domain Descriptor
MMM	MAC Management Message
NCP	Next Codeword Pointer
OCD	OFDM Channel Descriptor
SC-QAM	Single Carrier Quadrature Amplitude Modulation
PHY	Physical Layer
PLC	PHY Link Channel
QAM	Quadrature Amplitude Modulation
UCD	Upstream Channel Descriptor
US	Upstream

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